Description of the spin structure function g_1 at arbitrary x and arbitrary Q^2

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The explicit expressions describing the structure function g_1 at arbitrary x and Q^2 are obtained. In the first place, they combine the well-known DGLAP expressions for g_1 with the total resummation of leading logarithms of x, which makes possible to cover the kinematic region of arbitrary x and large Q^2. In order to cover the small-Q^2 region the shift $Q^2 \to Q^2 + \mu^2$ in the large-Q^2 expressions for g_1 is suggested and values of μ are estimated. The expressions obtained do not require singular factors x^{-a} in the fits for initial parton densities.

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I. INTRODUCTION

The goal of obtaining universal expressions describing the structure function g_1 at all x and Q^2 is an attractive task from both theoretical and phenomenological point of view. Until recently, the only theoretical instrument to describe g_1 was the Standard Approach (SA) which involves the DGLAP evolution equations[1] and standard fits[2] for the initial parton densities δq and δg. The fits are defined from phenomenological considerations at $x \sim 1$ and $Q^2 = \mu^2 \sim 1\text{GeV}^2$. The DGLAP equations are one-dimensional, they describe the Q^2 -evolution only, converting δq and δg into the evolved distributions Δq and Δg. The DGLAP equations are theoretically grounded in the kinematical the region **A** only:

$$\mathbf{A:} \qquad s > Q^2 \gg \mu^2, \qquad x \lesssim 1 \tag{1}$$

where we have denoted $s \equiv 2pq$, with p and q being the momenta of the initial hadron and photon respectively. This leaves the other kinematical regions uncovered. It is convenient to specify those regions as follows:

The small-x region **B**:

$$\mathbf{B:} \qquad s \gg Q^2 \gg \mu^2, \qquad x \ll 1 \tag{2}$$

and the small-Q^2 regions **C** and **D** :

$$\mathbf{C:} \qquad 0 \leq Q^2 \lesssim \mu^2, \qquad x \ll 1, \tag{3}$$

$$\mathbf{D:} \qquad 0 \leq Q^2 \lesssim \mu^2, \qquad x \lesssim 1. \tag{4}$$

As the matter of fact, the SA has been extended from Region **A** to the small-x Region **B**, though without any theoretical basis. The point is that after converting δq and δg into Δq and Δg with the DGLAP evolution equations, they should be evolved to the small-x region as well. The x -evolution is supposed to come from convoluting Δq and Δg with the coefficient functions C_{DGLAP}. However, in the leading order $C^{LO}_{DGLAP} = 1$; the NLO corrections account for one- or two- loop contributions and neglect higher loops. This is the correct approximation in the region **A** but becomes wrong in the Region **B** where contributions $\sim \ln^k(1/x)$ are large and should be accounted for to all orders in α_s. C_{DGLAP} do no include the total resummation of the leading logarithms of x (LL), so SA requires special fits for δq and δg. The general structure of such fits (see Refs. [2]) is as follows:

$$\delta q = N x^{-a} \varphi(x) \tag{5}$$

where N is a normalization constant; $a > 0$, so x^{-a} is singular when $x \to 0$ and $\varphi(x)$ is regular in x at $x \to 0$. In Ref. [3] we showed that the role of the factor x^{-a} in Eq. (5) is to mimic the total resummation of LL performed in

Refs [4, 5]. Similarly to LL, the factor x^{-a} provides the steep rise to g_1 at small x and sets the Regge asymptotics for g_1 at $x \to 0$, with the exponent a being the intercept. The presence of this factor is very important for extrapolating DGLAP into the region **B:** When the factor x^{-a} is dropped from Eq. (5), DGLAP stops to work at $x \lesssim 0.05$ (see Ref. [3] for detail). Accounting for the LL resummation is beyond the DGLAP framework, because LL come from the phase space not included in the DGLAP -ordering

$$\mu^2 < k_{1\,\perp}^2 < k_{2\,\perp}^2 < ... < Q^2 \tag{6}$$

for the ladder partons ($k_{2i\,\perp}$ are the transverse components of the ladder momenta k_i). LL can be accounted only when the ordering Eq. (6) is lifted and all $k_{i\,\perp}$ obey

$$\mu^2 < k_{i\,\perp}^2 < (p+q)^2 \approx (1-x)2pq \approx 2pq \tag{7}$$

at small x. Replacing Eq. (6) by Eq. (7) leads inevitably to the change of the DGLAP parametrization

$$\alpha_s^{DGLAP} = \alpha_s(Q^2) \tag{8}$$

by the alternative parametrization of α_s given by Eq. (14). This parametrization was obtained in Ref. [6] and was used in Refs. [4, 5] in order to find explicit expressions accounting for the LL resummation for g_1 in the region **B**. Obviously, those expressions require the non-singular fits for the initial parton densities. Let us note that the replacement of Eq. (6) by Eq. (7) brings a more involved μ -dependence of g_1. Indeed, Eq. (6) makes the contributions of gluon ladder rungs be infrared (IR) stable, with μ acting as a IR cut-off for the lowest rung and $k_{i\,\perp}$ playing the role of the IR cut-off for the $i+1$-rung. In contrast, Eq. (7) implies that μ acts as the IR cut-off for every rung.

The small-Q^2 Regions **C** and **D** are, obviously, beyond the reach of SA because DGLAP cannot be exploited here. Alternatively, in Refs. [7, 8] we obtained expressions for g_1 in the region **C** and proved that Region C can be described through the shift $Q^2 \to Q^2 + \mu^2$ in our large-Q^2 formulae. Combining these results with SA obtained in Ref. [3] makes it possible to describe g_1 in Region **D**. For the sake of simplicity, we present below formulae for g_1^{NS}, the non-singlet component of g_1 only.

II. DESCRIPTION OF g_1 IN THE REGION B

The total resummation of the double-logarithms (DL) and single- logarithms of x in the region **B** was done in Refs. [4, 5]. In particular, the non-singlet component, g_1^{NS} of g_1 is

$$g_1^{NS}(x, Q^2) = (e_q^2/2) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/x)^{\omega} C_{NS}(\omega) \delta q(\omega) \exp\big(H_{NS}(\omega) \ln(Q^2/\mu^2)\big) \ , \tag{9}$$

with new coefficient functions C_{NS},

$$C_{NS}(\omega) = \frac{\omega}{\omega - H_{NS}^{(\pm)}(\omega)} \tag{10}$$

and anomalous dimensions H_{NS},

$$H_{NS} = (1/2)\Big[\omega - \sqrt{\omega^2 - B(\omega)}\Big] \tag{11}$$

where

$$B(\omega) = (4\pi C_F(1+\omega/2)A(\omega) + D(\omega))/(2\pi^2) \ . \tag{12}$$

$D(\omega)$ and $A(\omega)$ in Eq. (12) are expressed in terms of $\rho = \ln(1/x)$, $\eta = \ln(\mu^2/\Lambda_{QCD}^2)$, $b = (33 - 2n_f)/12\pi$ and the color factors $C_F = 4/3$, $N = 3$:

$$D(\omega) = \frac{2C_F}{b^2 N} \int_0^{\infty} d\rho e^{-\omega\rho} \ln\big(\frac{\rho+\eta}{\eta}\big)\Big[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} \mp \frac{1}{\eta}\Big] \ , \tag{13}$$

$$A(\omega) = \frac{1}{b}\Big[\frac{\eta}{\eta^2+\pi^2} - \int_0^{\infty} \frac{d\rho e^{-\omega\rho}}{(\rho+\eta)^2+\pi^2}\Big]. \tag{14}$$

H_S and C_{NS} account for DL and SL contributions to all orders in α_s. Eqs. (14) and (13) depend on the IR cut-off μ through variable η. It is shown in Refs. [4, 5] that there exists an Optimal scale for fixing μ: $\mu \approx 1$ Gev for g_1^{NS} and $\mu \approx 5$ GeV for g_1^s. The arguments in favor of existence of the Optimal scale were given in Ref. [8]. Eq. (9) predicts that g_1 exhibits the power behavior in x and Q^2 when $x \to 0$:

$$g_1^{NS} \sim \left(Q^2/x^2\right)^{\Delta_{NS}/2}, \; g_1^S \sim \left(Q^2/x^2\right)^{\Delta_S/2} \tag{15}$$

where the non-singlet and singlet intercepts are $\Delta_{NS} = 0.42, \; \Delta_S = 0.86$ respectively. However the asymptotic expressions (15) should be used with great care: According to Ref. [3], Eq. (15) should not be used at $x \gtrsim 10^{-6}$. So, Eq. (9) should be used instead of Eq. (15) at available small x. Expressions accounting the total resummation of LL for the singlet g_1 in the region **B** were obtained in Ref. [5]. They are more complicated than Eq. (9) because involve two coefficient functions and four anomalous dimensions.

III. UNIFIED DESCRIPTION OF REGIONS A AND B

As was suggested in Ref. [3], the natural way to describe g_1 in the Regions A and B is to combine the small-x results with the DGLAP expressions for the coefficient functions and anomalous dimensions of g_1. In particular, g_1^{NS} is again given by Eq. (9), however with the new coefficient function $\widetilde{C}_{NS}$ and new anomalous dimension $\widetilde{H}_{NS}$:

$$\begin{aligned} \widetilde{C}_{NS} &= C_{NS} + C_{NS}^{DGLAP} - \Delta C_{NS} \\ \widetilde{H}_{NS} &= H_{NS} + \gamma_{NS}^{DGLAP} - \Delta H_{NS} \end{aligned} \tag{16}$$

where C_{NS} and H_{NS} are defined in Eqs. (10,11), C_{NS}^{DGLAP} and γ_{NS}^{DGLAP} are the DGLAP non-singlet coefficient function and anomalous dimension. The terms $\Delta C_{NS}, \; \Delta H_{NS}$ should be introduced to avoid the double counting. In the case when the DGLAP expressions are used in C_{NS}^{DGLAP} and γ_{NS}^{DGLAP} with the LO accuracy,

$$\Delta C_{NS} = 1, \quad \Delta H_{NS} = \frac{A(\omega)}{2\pi}\Big[\frac{1}{\omega} + \frac{1}{2}\Big] \tag{17}$$

They are the first terms of expansions of Eqs. (10,11) in the series in $A(\omega)$. In order to account for the NLO terms for C_{NS}^{DGLAP} and γ_{NS}^{DGLAP}, the next terms of the expansions should be included into ΔC_{NS} and ΔH_{NS}. When Eq. (16) is substituted into Eq. (9), we arrive at the description of g_1^{NS} covering both Regions **A** and **B**. Obviously, the main contribution to $\widetilde{C}_{NS}, \; \widetilde{H}_{NS}$ at Region **A** comes from their DGLAP components. On the contrary, the total resumation terms dominate at $x \ll 1$. When Eq. (16) is used, the initial parton densities should not include singular factors.

IV. DESCRIPTION OF g_1 IN THE REGIONS B AND C

Region **C** is defined in Eq. (3). It involves small Q^2, so there are no large contributions $\ln^k(Q^2/\mu^2)$ in this region. In other words, the DGLAP ordering of Eq. (6) does not make sense in the region **C** , which makes impossible exploiting DGLAP here. In contrast, Eq. (6) is not sensitive to the value of Q^2 and therefore the total resummation of LL does make sense in the region **C**. In Ref. [7] we suggested that the shift

$$Q^2 \to Q^2 + \mu^2 \tag{18}$$

would allow for extrapolating our previous results (obtained in Refs. [4, 5] for g_1 in the region **B**) into the region **C**. Then in Ref. [8] we proved this suggestion. Therefore, applying Eq. (18) to g_1^{NS} leads to the following expression for g_1^{NS} valid in the regions **B** and **C**:

$$g_1^{NS}(x+z, Q^2) = (e_q^2/2) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \Big(\frac{1}{x+z}\Big)^{\omega} C_{NS}(\omega) \delta q(\omega) \exp\left(H_{NS}(\omega) \ln\left((Q^2+\mu^2)/\mu^2\right)\right), \tag{19}$$

where $z = \mu^2/2pq$. Obviously, Eq. (19) reproduces Eq. (9) in the region **B**. Expression for g_1^S looks similarly but more complicated, see Refs. [7, 8] for detail. Let us notice that the idea of considering DIS in the small-Q^2 region through the shift Eq. (18) is not new. It was introduced by Nachtmann in Ref. [10] and used after that by many authors (see e.g. [11]), being based on different phenomenological considerations. On the contrary, our approach is based on the analysis of the Feynman graphs contributing to g_1. We also suggest that the following values for μ should be used: for the non-singlet component of g_1 $\mu = 1$ GeV and $\mu = 5.5$ GeV for the singlet g_1.

V. GENERALIZATION TO THE REGION D

The generalization of the results of Sect. IV to the Region **D** can easily be done with replacements

$$C_{NS} \to \widetilde{C}_{NS}, \qquad H_{NS} \to \widetilde{H}_{NS} \tag{20}$$

in Eq. (19), with $\widetilde{C}_{NS}$, $\widetilde{H}_{NS}$ defined in Eq. (16). So, we arrive at the final result: the expression for g_1 which can be used in the Regions **A,B,C,D** universally is

$$g_1^{NS}(x+z, Q^2) = (e_q^2/2) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \Big(\frac{1}{x+z}\Big)^{\omega} \widetilde{C}_{NS}(\omega)\delta q(\omega) \exp\big(\widetilde{H}_{NS}(\omega) \ln\big((Q^2+\mu^2)/\mu^2\big)\big). \tag{21}$$

We remind that the expressions for the initial parton densities in Eq. (21) should not contain singular terms because the total resummation of leading logarithms of x is explicitly included into $\widetilde{C}_{NS}$ and $\widetilde{H}_{NS}$.

VI. PREDICTION FOR THE COMPASS EXPERIMENTS

The COMPASS collaboration now measures the singlet g_1^S at $x \sim 10^{-3}$ and $Q^2 \lesssim 3$ GeV^2, i.e. in the kinematic region beyond the reach of DGLAP. However, our formulae for g_1^{NS} and g_1^S obtained in Refs. [7, 8] cover this region. Although expressions for singlet and non-singlet g_1 are different, with formulae for the singlet being much more complicated, we can explain the essence of our approach, using Eq. (19) as an illustration. According to results of [5], $\mu \approx 5$ GeV for g_1^S, so in the COMPASS experiment $Q^2 \ll \mu^2$. It means, $\ln^k(Q^2+\mu^2)$ can be expanded into series in Q^2/μ^2, with the first term independent of Q^2:

$$g_1^S(x+z, Q^2, \mu^2) = g_1^S(z, \mu^2) + \sum_{k=1} (Q^2/\mu^2)^k E_k(z) \tag{22}$$

where $E_k(z)$ account for the total resummation of LL of z and

$$g_1^S(z, \mu^2) = (< e_q^2/2 >) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} \big(1/z\big)^{\omega} \big[C_S^q(\omega)\delta q(\omega) + C_S^g(\omega)\delta g(\omega)\big], \tag{23}$$

so that $\delta q(\omega)$ and $\delta g(\omega)$ are the initial quark and gluon densities respectively and $C_S^{q,g}$ are the singlet coefficient functions. Explicit expressions for $C_S^{q,g}$ are given in Refs. [5, 7]. Therefore, we can makes the following predictions easy to be checked by COMPASS:

A. Prediction 1

In the whole COMPASS range $0 \lesssim Q^2 \lesssim 3$ GeV^2, the singlet g_1 does not depend on x regardless of the value of x.

B. Prediction 2

Instead of studying experimental the x-dependence of g_1^S, it would be much more interesting to investigate its dependence on $2pq$ because it makes possible to estimate the ratio $\delta g/\delta q$ (see Ref. [7] for detail).

VII. REMARK ON THE HIGHER TWISTS CONTRIBUTIONS

In the region **B** one can expand terms $\sim (Q^2+\mu^2)^k$ in Eq. (19) into series in $(\mu^2/Q^2)^n$ and represent $g_1^{NS}(x+z, Q^2, \mu^2)$ as follows:

$$g_1^{NS}(x+z, Q^2, \mu^2) = g_1^{NS}(x, Q^2/\mu^2) + \sum_{k=1} (\mu^2/Q^2)^k T_k \tag{24}$$

where $g_1^{NS}(x, Q^2/\mu^2)$ is given by Eq. (9); for explicit expressions for the factors T_k see Ref. [8]. The power terms in the rhs of Eq. (24) look like the power $\sim 1/(Q^2)^k$ -corrections and therefore the lhs of Eq. (24) can be interpreted as the total resummation of such corrections. These corrections are of the perturbative origin and have nothing in common with higher twists contributions ($\equiv HTW$). The latter appear in the conventional analysis of experimental date on the Polarized DIS as a discrepancy between the data and the theoretical predictions, with $g_1^{NS}(x, Q^2/\mu^2)$ being given by the Standard Approach:

$$g_1^{NS\ exp} = g_1^{NSSA} + HTW\ . \tag{25}$$

Confronting Eq. (25) to Eq. (24) leads to an obvious conclusion: In order estimate genuine higher twists contributions to g_1^{NS}, one should account, in the first place, for the perturbative power corrections predicted by Eq. (24); otherwise the estimates cannot be reliable. It is worth mentioning that we can easily explain the empirical observation made in the conventional analysis of experimental data: The power corrections exist for $Q^2 > 1$ GeV^2 and disappear when $Q^2 \to 1$ GeV^2. Indeed, in Eq. (24) $\mu = 1$ GeV , so the expansion in the rhs of Eq. (24) make sense for $Q^2 > 1$ GeV^2 only; at smaller Q^2 it should be replaced by the expansion of Eq. (19) in $(Q^2/\mu^2)^n$.

VIII. CONCLUSION

The extrapolation of DGLAP from the standard Region **A** to the small-x Region B involves necessarily the singular fits for the initial parton densities without any theoretical basis. On the contrary, the resummation of the leading logarithms of x is the straightforward and most natural way to describe g_1 at small x. Combining this resummation with the DGLAP results leads to the expressions for g_1 which can be used at large Q^2 and arbitrary x (Regions **A** and **B**), leaving the initial parton densities non-singular. Then, incorporating the shift of Eq. (18) into these expressions allows us to describe g_1 in the small-Q^2 regions (Regions **C** and **D**) and to write down Eq. (21) describing g_1 at the Regions **A,B,C,D.** We have used it for studying the g_1 singlet at small Q^2 which is presently investigated by the COMPASS collaboration. It turned out that g_1 in the COMPASS kinematic region depends on $z = \mu^2/2pq$ only and practically does not depend on x, even at $x \ll 1$. Numerical calculations show that the sign of g_1 is positive at z close to 1 and can remain positive or become negative at smaller z, depending on the ratio between δg and δq. To conclude, let us notice that extrapolating DGLAP into the small-x region, although it could provide a satisfactory agreement with experimental data, leads to various wrong statements, or misconceptions. We enlisted the most of them in Ref. [9]. Below we mention one important wrong statements not included in Ref. [9]:

Misconception: *The impact of the resummation of leading logarithms of x on the small-x behavior of g_1 is small.*

This statement appears when the resummation is combined with the DGLAP expressions, similarly to Eq. (16), and at the same time the fits for the initial parton densities contain singular factors like the one in Eq. (5). Such a procedure is inconsistent and means actually a double counting of the logarithmic contributions: the first implicitly, through the fits, and the second in explicit way. It also affects the small-x asymptotics of g_1, leading to the incorrect values of the intercepts of g_1 (see Ref. [3] for more detail).

IX. ACKNOWLEDGEMENT

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